Midterm Exam Sol.
2014-2015

1.

$$C_n = \frac{1}{T_0}\int^{T_0} g_p(t) \cdot e^{-Jn\omega_0 t}\, dt$$

$$= \frac{A}{T_0}\int_0^{T_0/2} e^{-Jn\omega_0 t}\, dt$$

$$= \frac{A}{T_0} \cdot \left. \frac{e^{-Jn\omega_0 t}}{-Jn\omega_0} \right|_0^{T_0/2}$$

$$= \frac{A}{-Jn\omega_0 T_0} \cdot \left[e^{-Jn\omega_0 T_0/2} - e^{0} \right]$$

$$= \frac{A}{-Jn(2\pi)} \cdot \left[e^{-Jn\pi} - 1 \right]$$

$$= \frac{A}{-2Jn\pi} \cdot \left[e^{-Jn\pi} - 1 \right]$$

$$\circ \quad e^{-Jn\pi} = \cos(n\pi) - J \underbrace{\sin(n\pi)}_{\to 0}$$

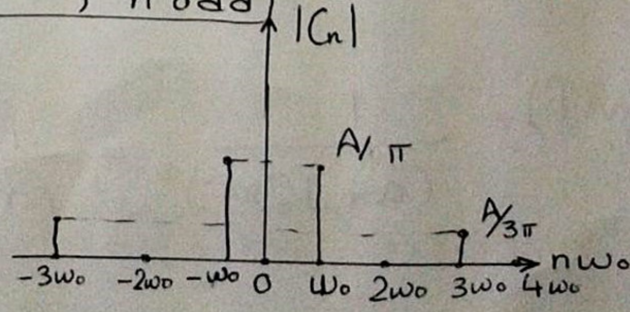
$$= (-1)^n$$

$$C_n = \frac{A}{-2Jn\pi} \left[(-1)^n - 1 \right] \rightarrow \begin{cases} C_n = 0, & n \text{ even} \\ C_n = \dfrac{A}{Jn\pi}, & n \text{ odd} \end{cases}$$

$$\therefore \boxed{|C_n| = \frac{A}{n\pi}, \quad n \text{ odd}}$$

$$\therefore \quad g_p(t) = \sum C_n \cdot e^{+Jn\omega_0 t}$$

$$\boxed{g_p(t) = \sum_{n=-\infty}^{\infty} \frac{A}{Jn\pi} \cdot e^{+Jn\omega_0 t}, \quad n \text{ odd}}$$



2. a.

$$x(t) = 2\,\text{rect}\left(\frac{t-5}{10}\right) + 8\sin(8\pi t)$$

$$= 20\,\text{Sinc}(10f)\cdot e^{-J2\pi f(5)} + \frac{8}{2j}\left[\delta(f-4) - \delta(f+4)\right]$$

b. $g(t) = \frac{1}{2}\delta(t+\frac{1}{4}) + \frac{1}{2}\delta(t-1/4)$.

$$= \frac{1}{2}\left[e^{j2\pi f(1/4)} + e^{-j2\pi f(1/4)}\right]$$

$$= \frac{e^{j\pi f/2} + e^{-j\pi f/2}}{2} = \cos(\pi f/2)$$

c. $w(t) = t\,e^{-at}\,u(t)$

$W(f) = ?$

$\cdot\ e^{-at}\cdot u(t) \rightleftharpoons \dfrac{1}{a+J2\pi f}$

$\because\ (-j2\pi t)\,g(t) \rightleftharpoons \dfrac{dG(f)}{df}$

$\to w(t)$

$\therefore\ t\cdot g(t) \rightleftharpoons \dfrac{dG(f)}{df}\cdot\dfrac{1}{-J2\pi} \to W(f)$

$g(t) = e^{-at}\cdot u(t)\ ,\quad G(f) = \dfrac{1}{a+J2\pi f}$

$\dfrac{dG(f)}{df} =$ (المقام * مشتقة البسط − البسط * مشتقة المقام) / المقام²

$$= \frac{\cancel{(a+J2\pi f)} * 0 - (J2\pi)}{(a+J2\pi f)^2}$$

$$\therefore\ W(f) = \frac{1}{(a+J2\pi f)^2} = \frac{1}{(a+J\omega)^2}$$

4. $m(t) = \sin(2000\pi t) + 2\cos(4000\pi t)$

$c(t) = 100\cos(2\pi f_c t)$, $f_c = 1\,MHz \longrightarrow k_a = 0.01$

DSBTC

a) Find & Sketch the Spectrum.

$$S(t) = A_c(1 + k_a \cdot m(t))\cos(2\pi f_c t)$$

$$= 100\,(1 + \underset{\mu_1}{0.01}\sin(2000\pi t) + \underset{\mu_2}{0.02}\cos(4000\pi t)).$$
$$\cos(2\pi \cdot 10^6 t)$$

$$= 100\cos(2\pi \cdot 10^6 t) + \frac{1}{2}\left[\sin(2\pi(-10^6 + 1000))t + \sin(2\pi(10^6 + 1000))t\right.$$
$$+ \left[\cos(2\pi(10^6 - 2000)t) + \cos(2\pi(10^6 + 2000)t)\right].$$

$$\therefore S(f) = \frac{100}{2}\left[\delta(f - 10^6) + \delta(f + 10^6)\right] + \frac{1}{4j}\left[\delta(f - (-10^6 + 1000))\right.$$
$$- \delta(f + (-10^6 + 1000)) + \delta(f - (10^6 + 1000)) - \delta(f + (10^6 + 1000)$$
$$+ \frac{1}{2}\left[\delta(f - (10^6 - 2000)) + \delta(f + (10^6 - 2000)) + \delta(f - (10^6 + 2000))\right.$$
$$+ \delta(f + (10^6 + 2000))\right]$$

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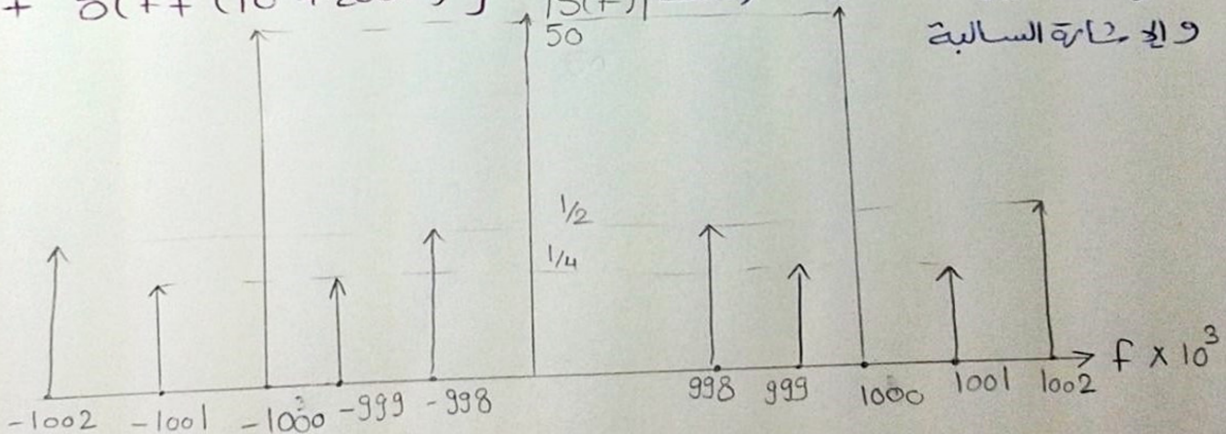

b) $P_c = ?$ $\quad P_{DSB} = ?$

$$P_c = \frac{A_c^2}{2} = \frac{100^2}{2} = 5000 \text{ watt}$$

$$P_{DSB} = \frac{A_c^2 \mu_t^2}{4} = 1.21 \text{ watt}$$

$$\text{Percentage} = \frac{P_c}{P_c + P_{DSB}} * 100\%$$
$$= \frac{5000}{5000 + 1.21} * 100\%$$
$$= 99.975\%$$

c) $\mu_t = ?$ $\quad \mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{0.01^2 + 0.02^2} = 0.022$